MAT2006: Elementary Real Analysis
Homework 8

W. Lee.

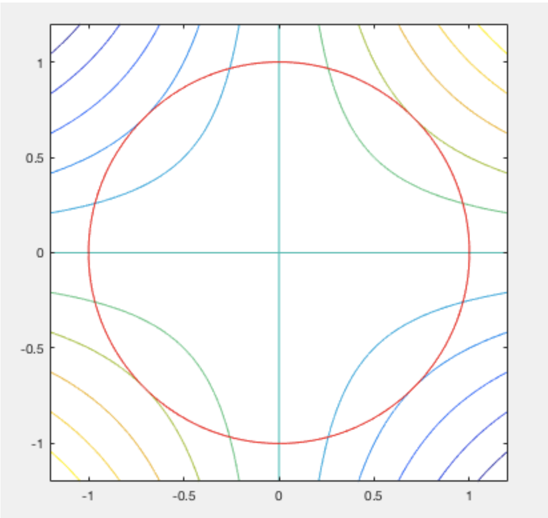
Due date: Dec. 16, 2018

Question 8.7-4. In the plane $\mathbb{R}^2$ with Cartesian coordinates x, y construct the level curves of the function $f(x, y) = xy$ and the curve

$$S = \{(x, y) \in \mathbb{R}^2 \,|\, x^2 + y^2 = 1\}$$

Using the resulting picture, carry out a complete investigation of the problem of extrema of the function $f\Big|_S$.

The level curve and one-dimensional sphere S is as follows,



When $x > 0, y > 0$ or $x < 0, y < 0$, the further the curve away from origin, the higher level it represents. We consider the curves that intersect with the sphere S. The maximum value of f restricting to S is the curve that is **tangent to the sphere** $\boldsymbol{S}$. The points can be easily calculated, which are $(\sqrt{2}/2, \sqrt{2}/2)$ and $(-\sqrt{2}/2, -\sqrt{2}/2)$. Also $f(x, y) = xy = 1/2$ at this two points. We don't have level curve that represents larger value than this, because the larger curve than this is farther and will have no common point with the circle.

Similarly, we can show that when $x > 0, y < 0$ or $x < 0, y > 0$, $(-\sqrt{2}/2, \sqrt{2}/2)$ and $(\sqrt{2}/2, -\sqrt{2}/2)$ is the points where $f(x, y)$ assumes its minimum value, which is $xy = -1/2$. You could use Lagrange Multiplier to calculate the answer more rigorously.

This is the end of MAT2006. Maybe you had a ball with him, or you just hate his guts. How much knowledge you learned or how many techniques you acquired doesn't really matter; it is the change of the way you think and the process of maturing that count. Wish you sail through the finals.

Life is hard, but we will still Live on .